Convert the polar equation $r^2 = 2 - 5\sin 2\theta$ to rectangular, and simplify as shown in the website handout. **SCORE:** ____ / 4 PTS

NOTE: Your final answer should NOT have fractions, radicals, nor negative or fractional exponents.

$r^2 = 2 - 5(2\sin\theta\cos\theta)$ (1)

$r^2 = 2 - 10\sin\theta\cos\theta$

$r^2 = 2 - 10\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$ ($\frac{1}{2}$) ($\frac{1}{2}$)

$r^2 = 2 - \frac{10xy}{r^2}$

$r^4 = 2r^2 - 10xy$ (1)

(1) $(x^2+y^2)^2 = 2(x^2+y^2) - 10xy$

$(x^2+y^2)^2 = 2x^2 - 10xy + 2y^2$

SEE ALTERNATE SOLUTION BELOW

GRADE AGAINST ONLY 1 VERSION

Convert the polar equation $r^2 = 2 - 5\sin 2\theta$ to rectangular, and simplify as shown in the website handout. **SCORE:** _____ / 4 PTS

NOTE: Your final answer should NOT have fractions, radicals, nor negative or fractional exponents.

$r^2 = 2 - 5(2\sin\theta\cos\theta)$ (1)

$r^2 = 2 - 10\sin\theta\cos\theta$

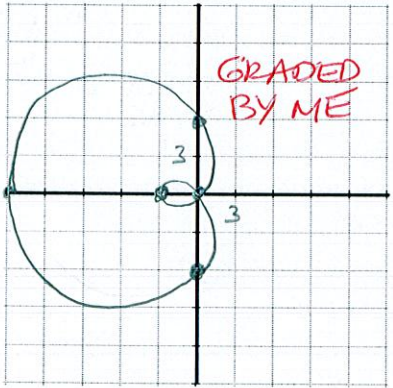
$r^4 = 2r^2 - 10(r\sin\theta)(r\cos\theta)$ (2)

$(x^2+y^2)^2 = 2(x^2+y^2) - 10yx$ (1)

$(x^2+y^2) = 2x^2 - 10xy + 2y^2$

Consider the graph of the polar equation $r = 6 - 9\cos\theta$. $0 < \left|\frac{6}{-9}\right| < 1$

SCORE: _____ / 6 PTS



[a] Fill in the blanks.

MUST HAVE BOTH WORDS FOR ANY POINTS

[i] The shape of the graph is a/an (1) LIMAÇON WITH LOOP .

[ii] The graph DOES (½) pass through the pole.
does / does not

[iii] Find the **rectangular coordinates** of the

θ	r
0	-3
$\frac{\pi}{2}$	6
π	15
$\frac{3\pi}{2}$	6

x – intercept(s)

$x =$ (½) -3 AND (1) -15 AND 0
(rectangular coordinates)

y – intercept(s)

$y =$ (½) -6 AND (½) 6 AND 0
(rectangular coordinates)

[b] Sketch the graph on the grid provided above. **You must provide a scale for the axes & plot all points from part [a][iii] above.**

The graph of the polar equation $r = 4\sqrt{3} - 8\cos 3\theta$ is symmetric only over the polar axis. **SCORE: _____ / 5 PTS**

[a] What is the minimum interval of the graph you need to plot first, before using reflections to draw the rest of the graph ?

$[0, \pi]$ (½)

[b] **Algebraically** find the angles in the minimum interval in part [a] at which the graph goes through the pole.
NOTE: You will NOT receive credit for just plugging in numbers to guess the answers.

$0 = 4\sqrt{3} - 8\cos 3\theta$ (1)

$0 \le \theta \le \pi$

$0 \le 3\theta \le 3\pi$

$\frac{\sqrt{3}}{2} = \cos 3\theta$ (½)

$3\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$ (1½)

$\theta = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}$ (1½)

The $(r, \pi - \theta)$ test is the only test that indicates that the graph of $r = 4 + 2\sin 3\theta$ is symmetric. **SCORE: _____ / 6 PTS**

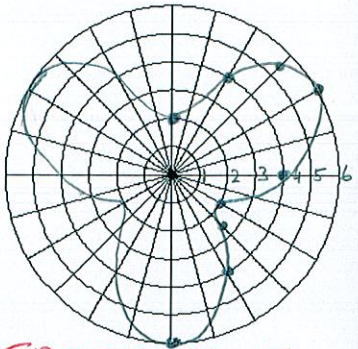
The graph passes through the polar points $(4, 0), (6, \frac{\pi}{6}), (5.4, \frac{\pi}{4}), (4, \frac{\pi}{3})$ and $(2, \frac{\pi}{2})$.

[a] What is the minimum interval of the graph you need to plot first, before using reflections to draw the rest of the graph ?

SYM OVER $\theta = \frac{\pi}{2}$ $\longrightarrow$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (1)

[b] Find the value of r (**rounded to 1 decimal place**) for all other common angles in the minimum interval in part [a].
You do **NOT** need to find r for the angles given in the original problem description.
NOTE: You do NOT need to show work, only answers.

θ		r
$-\frac{\pi}{6}$	$\frac{1}{2}$	$2 = 4 + 2\sin(-\frac{\pi}{2}) = 4 + 2 \cdot -1$
$-\frac{\pi}{4}$	$\frac{1}{2}$	$2.6 \approx 4 + 2\sin(-\frac{3\pi}{4}) = 4 + 2 \cdot \frac{-\sqrt{2}}{2} = 4 - \sqrt{2}$
$-\frac{\pi}{3}$	$\frac{1}{2}$	$4 = 4 + 2\sin(-\pi) = 4 + 2 \cdot 0$
$-\frac{\pi}{2}$	$\frac{1}{2}$	$6 = 4 + 2\sin(-\frac{3\pi}{2}) = 4 + 2 \cdot 1$



GRADED BY ME

[c] Sketch the graph on the grid provided above. **You must provide a scale for the polar axis & plot all points given and found.**

Consider the graph of the polar equation $r = 5 - 3\sin 4\theta$.

SCORE: _____ / 5 PTS

NOTE: The $(-r, \pi-\theta)$, $(r, \pi-\theta)$ and $(r, -\theta)$ tests do NOT show that the graph is symmetric

POLAR AXIS $\theta = \frac{\pi}{2}$ POLAR AXIS

Using the information above, and the tests and shortcuts shown in lecture, test if the graph is symmetric over the pole, the polar axis, and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE $(r, \pi+\theta)$:

① $r = 5-3\sin 4(\pi+\theta)$

$r = 5-3\sin(4\pi+4\theta)$

$r = 5-3(\sin 4\pi^{0}\cos 4\theta + \cos 4\pi \sin 4\theta)$

① $r = 5-3\sin 4\theta$

$\theta = \frac{\pi}{2}$ $(-r, -\theta)$:

$-r = 5-3\sin 4(-\theta)$ ①

$-r = 5+3\sin 4\theta$

$r = -5-3\sin 4\theta$ ½

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

GRADED BY ME

Fill in the blanks. **NOTE: All answers for parts [a] and [b] must be positive.** (1) **SCORE:** _____ / 4 PTS

[a] The polar co-ordinates $(-2, -\frac{13\pi}{8})$ correspond to the same point as the polar co-ordinates $(2, \underline{\frac{11\pi}{8}})$ and $(-2, \underline{\frac{3\pi}{8}})$. (½)

$\frac{3\pi}{8} + \pi$ $\quad \frac{-13\pi}{8} + 2\pi$

[b] The point with rectangular co-ordinates $(-\sqrt{15}, 3\sqrt{5})$ has polar co-ordinates $(\underline{2\sqrt{15}}, \underline{\frac{2\pi}{3}})$. (½) (1)

$\sqrt{15+45} = \sqrt{60}$

[c] The point with polar co-ordinates $(4, -\frac{4\pi}{3})$ has rectangular co-ordinates $(\underline{-2}, \underline{2\sqrt{3}})$. (½) (½)

$4\cos(-\frac{4\pi}{3}) = 4 \cdot -\frac{1}{2}$ $\quad 4\sin(-\frac{4\pi}{3}) = 4 \cdot \frac{\sqrt{3}}{2}$

$\pi + \tan^{-1}(\frac{-3}{\sqrt{3}})$

$= \pi + \tan^{-1}(-\sqrt{3})$

$= \pi - \frac{\pi}{3}$